Teaching Fractions Using Whole - To- Part Semantic Diagrams

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Visually Equivalent Fraction Cards

Oftentimes, students are able to perform conceptually-based shading tasks that link a semiconcrete model with a fraction. These shading activities involve a whole-to-part procedure. Students are required to view a whole as a collection of equal parts, then shade a subset of them. A problem arises when students are asked to create an equivalent form of a fraction e.g., \( \frac{1}{2} = \frac{1}{6} \)? Procedurally, the student is asked to make both components of \( \frac{1}{2} \), three times bigger. This sets the stage for a discrepant event to occur. On one hand, the student is being asked to make an equivalent fraction, on the other, he is asked to create a model that is (concretely) three times larger.

Consider the semiconcrete models below. From left to right are models of \( \frac{1}{2} \), then \( \frac{3}{6} \). Though procedurally equivalent and proportional, these representations present conflicting information. When conceptual models fail to generalize to multistep procedures, students lose the ability to apply learned concepts to their solution process. As a result, students abandon their conceptual understanding in favor of a rote procedural repertoire.

\[
\frac{1}{2} = \frac{3}{6} = \frac{1}{2} = \frac{3}{6} \, ?
\]

Model the Addition of Fractions With Uncommon Denominators Conceptually and Procedurally Using Whole-To-Part Methodology.

Use the enclosed graphic organizer cards to develop an integrated conceptual and procedural understanding of fraction equivalence. Students are led to acknowledge the concept of fraction equivalence from whole-to-part. They are prompted to shade a quantity of specific attributes (e.g., paws, legs or clock numbers) to represent a given fraction that is expressed in lowest terms (simplified). For instance, shade 1/2 of the dog’s paws: shade two of the four paws along the dog’s axis of symmetry. This task is instrumental in establishing a durable conceptual model of fraction equivalency. Later, the procedural method of producing an equivalent fraction is presented in conjunction with this visual/semantic model.

First, have students shade paws, legs and clock sections with a highlighter as prescribed by the fraction printed on each card. Generally, divide along the recommended line(s) of symmetry. Shade portions of the clock in a clockwise manner, starting at the 12 position.

\[
\frac{1}{2}
\]

\[
\frac{1}{3}
\]

* Note- there is no shading on the dog or spider \( \frac{1}{3} \) cards as 4 and 8 are not divisible by 3.

Next, develop the students’ ability to label the shaded cards using fractions in their instrumental form. For example, the students must practice recognizing that the ant card that is marked 1/2 may also be named 3/6, corresponding to the three shaded legs of the six-legged ant. Emphasize that the denominator (circled) represents the quantity of attributes (legs) needed to make one whole. The numerator (boxed) represents the number of attributes that are described as being present, or shaded.

E.g., 3/6 of an ant:

There are three shaded legs: \( \frac{3}{3} \)

It takes six legs to make one whole ant: \( \frac{6}{6} \)

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All Students Respond Activity: Equivalent Fractions- Whole-To-Part
Have students place all of their dog cards in a row, ant cards in the next row, and so forth. Ask the students to hold up the card that represents the fraction that is read to them. Choose fractions that are in their instrumental (unsimplified) form. For instance, “Hold up 4/8.” The 8 in the denominator should prompt them to select a spider card. After all students have responded by holding-up a card, choose a student that has responded correctly by holding up the spider card labeled 1/2. Acknowledge that they are correct. Ask that student to explain their solution process. This “all students respond” activity provides processing time for all students and benefits students with expressive language deficits. It also provides a relatively safe- or risk-free method of responding as the teacher will never call on a student who has made an error.

Ask the chosen student to name the fraction that is written on the card that they have accurately selected (1/2). Write 1/2 = 4/8 on the board. Show how to change 1/2 to 4/8 by multiplying the numerator and denominator by 4/4 (or 1). Develop the idea that multiplying by a fraction equivalent to 1 changes the way that the shaded material is labeled, but does not alter the quantity of shaded attributes. For instance, 1 of a spider’s 2 sides (1/2) can also be described as 4 of a spider’s 8 legs (4/8).

Fraction Addition- Whole-To-Part
Many students grapple for a reason for renaming fractions to equivalent forms with common denominators before adding them. Unable to employ semantic reasoning skills to the process, students resort to performing the procedure devoid of context. They are unable to attach any relational understanding to similar problems or validate the accuracy of their work product. Students should be led to understand the problem by presenting the fractions to be added within the context of a graphic organizer that represents their common denominator.

Use the cards to model the addition of fractions with uncommon denominators. Choose the problem by using the graphic organizer that will represent the common denominator of the two fractions. For instance, “Add 1/2 + 1/6.” The students should select two ant cards: the 1/2 card with three legs shaded, and the 1/6 card with one leg shaded. Ant cards are selected as both fractions may be represented by that organizer. Never mix graphic organizers.

Ask the students to place the cards one above the other, as shown in the diagram below. Have them name the total number of legs that are shaded on the two cards: 4. So, 4/6 of the legs are shaded. This creates an accurate semantic-based solution to the addition problem. Next, present the same problem on the board using the labels on the cards. Show how to rename the 1/2 as 3/6 using the shaded ant card. This activity is repeated with other fraction problems. Students demonstrate understanding by first using cards, then setting-up and solving corresponding fraction addition problem using the conventional renaming and adding procedure.

<table>
<thead>
<tr>
<th>Graphic Organizer</th>
<th>Semantic label</th>
<th>Problem</th>
<th>Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Graphic Organizer]</td>
<td>3 legs</td>
<td>( \frac{1}{2} = \frac{3}{6} )</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Graphic Organizer]</td>
<td>1 leg</td>
<td>( \frac{1}{6} = \frac{1}{6} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4 legs</td>
</tr>
</tbody>
</table>
Word Box
Wholes: dog, ant, spider, clock
Parts: paws, legs, hours

Circle the picture that shows an example of \(\frac{1}{4}\)

Describe the fraction that you chose:
# Parts you have shaded =
# Parts to make 1 whole =

Circle the picture that shows an example of \(\frac{1}{6}\)

Describe the fraction that you chose:
# Parts you have shaded =
# Parts to make 1 whole =

Circle the picture that shows an example of \(\frac{1}{8}\)

Describe the fraction that you chose:
# Parts you have shaded =
# Parts to make 1 whole =

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Circle the picture that shows an example of \( \frac{3}{4} \):

- Dog
- Spider
- Clock

Describe the fraction that you chose:

- # Parts you have shaded = __________
- # Parts to make 1 whole = __________

Circle the picture that shows an example of \( \frac{6}{6} \):

- Dog
- Spider
- Clock

Describe the fraction that you chose:

- # Parts you have shaded = __________
- # Parts to make 1 whole = __________

Circle the picture that shows an example of \( \frac{3}{12} \):

- Dog
- Spider
- Clock

Describe the fraction that you chose:

- # Parts you have shaded = __________
- # Parts to make 1 whole = __________

Word Box

Wholes: dog, ant, spider, clock
Parts: paws, legs, hours
Circle the picture that shows an example of:

[Image 1] 3
[Image 2] 8

# Parts you have shaded = [ ]

Describe the fraction that you chose:

# Parts to make 1 whole = [ ]

---

Circle the picture that shows an example of:

[Image 3] 4
[Image 4] 6

# Parts you have shaded = [ ]

Describe the fraction that you chose:

# Parts to make 1 whole = [ ]

---

Circle the picture that shows an example of:

[Image 5] 4
[Image 6] 12

# Parts you have shaded = [ ]

Describe the fraction that you chose:

# Parts to make 1 whole = [ ]
Circle the picture that shows an example of:

\[ \frac{3}{4} \]

Describe the fraction that you chose:

\[ \# \text{ parts you have shaded} = \] \[ \# \text{ parts to make 1 whole} = \]

\[ \frac{4}{8} \]

Describe the fraction that you chose:

\[ \# \text{ parts you have shaded} = \] \[ \# \text{ parts to make 1 whole} = \]

\[ \frac{6}{12} \]

Describe the fraction that you chose:

\[ \# \text{ parts you have shaded} = \] \[ \# \text{ parts to make 1 whole} = \]
Write a fraction to describe the shaded portion of each picture.

Number that you have

Number to make 1 whole

Paws

Fingers

Legs

Hours

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Write a fraction to describe the shaded portion of each picture.

Number that you have → \[ \frac{\text{Paws}}{\text{Circle}} \]
Number to make 1 whole → \[ \frac{\text{Fingers}}{\text{Circle}} \]

- Paws: \[ \frac{\text{Part}}{\text{Whole}} \]
- Fingers: \[ \frac{\text{Part}}{\text{Whole}} \]
- Legs: \[ \frac{\text{Part}}{\text{Whole}} \]
- Hours: \[ \frac{\text{Part}}{\text{Whole}} \]
Type I Fraction Addition
Shade 1 paw.

Shade 3 paws.

Add the shaded paws.
Shade the paws:

\[ \frac{1}{4} + \frac{2}{4} = \frac{3}{4} \]

Total Paws Shaded:

Add the shaded paws.
Shade the paws:

\[ \frac{3}{4} + \frac{1}{4} = \frac{4}{4} \]

Total Paws Shaded:

Add the shaded paws.
Shade the paws:

\[ \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \]

Total Paws Shaded:

Add the shaded paws.
Shade the paws:

\[ \frac{2}{4} + \frac{2}{4} = \frac{4}{4} \]

Total Paws Shaded:
Type I Fraction Addition

Shade 1 leg.

Shade the 2 front legs.

Shade the 3 left legs.

Shade the legs:

\[
\begin{array}{c}
\frac{1}{6} + \frac{2}{6} = \frac{3}{6} \\
\text{Total legs} \\
\text{Shaded:}
\end{array}
\]

\[
\begin{array}{c}
\frac{2}{6} + \frac{3}{6} = \frac{5}{6} \\
\text{Total legs} \\
\text{Shaded:}
\end{array}
\]

\[
\begin{array}{c}
\frac{3}{6} + \frac{3}{6} = \frac{6}{6} \\
\text{Total legs} \\
\text{Shaded:}
\end{array}
\]

\[
\begin{array}{c}
\frac{1}{6} + \frac{1}{6} = \frac{2}{6} \\
\text{Total legs} \\
\text{Shaded:}
\end{array}
\]
Type I Fraction Addition

Shade 1 leg.  Shade the 2 front legs.  Shade the 3 left legs.

Shade the legs:

\[
\begin{array}{c}
\quad \quad \quad \quad \quad \quad \\
\quad \quad \quad \quad \quad \quad \\
\quad \quad \quad \quad \quad \quad \\
\quad \quad \quad \quad \quad \quad \\
\end{array}
\]

Total legs Shaded:

\[
\begin{array}{c}
\quad \quad \quad \quad \quad \quad \\
\quad \quad \quad \quad \quad \quad \\
\quad \quad \quad \quad \quad \quad \\
\quad \quad \quad \quad \quad \quad \\
\end{array}
\]
Type I Fraction Addition

Shade 1 leg. Shade the 2 front legs.

Shade the 3 left legs.

Shade the legs:

\[
\begin{align*}
\text{Shade the legs:} & & \frac{1}{6} + \frac{1}{6} \\
\text{Total legs} & & \frac{6}{6}
\end{align*}
\]

Total legs Shaded:

Shade the legs:

\[
\begin{align*}
\text{Shade the legs:} & & \frac{5}{6} + \frac{1}{6} \\
\text{Total legs} & & \frac{6}{6}
\end{align*}
\]

Total legs Shaded:

Shade the legs:

\[
\begin{align*}
\text{Shade the legs:} & & \frac{2}{6} + \frac{3}{6} \\
\text{Total legs} & & \frac{6}{6}
\end{align*}
\]

Total legs Shaded:

Shade the legs:

\[
\begin{align*}
\text{Shade the legs:} & & \frac{4}{6} + \frac{1}{6} \\
\text{Total legs} & & \frac{6}{6}
\end{align*}
\]

Total legs Shaded:

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Type I Fraction Addition

Shade 1 leg.

Shade the 2 front legs. (1 pair of legs.)

Shade the 4 right legs:

Shade the legs:

Shade the legs:

Shade the legs:

Shade the legs:

Total legs Shaded:

Total legs Shaded:

Total legs Shaded:

Total legs Shaded:

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Type I Fraction Addition

Shade 1 leg.

Shade the 2 front legs. (1 pair of legs.)

Shade the legs:

\[
\begin{align*}
\text{Shade the legs:} & \quad \frac{3}{8} + \frac{1}{8} = \frac{4}{8} \\
\text{Total legs} & \quad \frac{4}{8} \\
\text{Shaded:} & \quad \frac{4}{8}
\end{align*}
\]

Shade the 4 right legs:

Shade the legs:

\[
\begin{align*}
\text{Shade the legs:} & \quad \frac{5}{8} + \frac{1}{8} = \frac{6}{8} \\
\text{Total legs} & \quad \frac{6}{8} \\
\text{Shaded:} & \quad \frac{6}{8}
\end{align*}
\]

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Write a fraction to describe the shaded portion of each picture, then rename the fraction.

Number that you have

Number to make 1 whole

- Sides

- Paws

- Hands

- Fingers

- Pairs of legs

- Legs

- Sides

- Legs

- Quarters

- Hours
Write a fraction to describe the shaded portion of each picture, then rename the fraction.

Number that you have

Number to make 1 whole

Sides = Paws

Hands = Fingers

Pairs of legs = Legs

Sides = Legs

Halves = Hours
Write a fraction to describe the shaded portion of each picture, then rename the fraction.

Number that you have

Number to make 1 whole
Introduction to Equivalent Fractions

Select the picture(s) that let you shade:

1/12

Select the picture(s) that let you shade:

1/2

Select the picture(s) that let you shade:

5/12

Select the picture(s) that let you shade:

1/4
Introduction to Equivalent Fractions

Select the picture(s) that let you shade:

7
12

Select the picture(s) that let you shade:

1
2

Select the picture(s) that let you shade:

1
4

Select the picture(s) that let you shade:

3
4

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Renaming Fractions

Choose a card to match the fraction provided. Use the picture to help rename the fraction. Write the multiplication rule that renames the fraction.

Place cards 1/2, 1/3, 2/3 and 1/6 here, one at a time.

1/2 = 1 x 3 = 3

1/3 = 1 x 2 = 2

2/3 = x

1/6 = x

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Renaming Fractions

Choose a card to match the fraction provided. Use the picture to help rename the fraction. Write the multiplication rule that renames the fraction.

Place card here

Use the same rule:

\[
\begin{align*}
\frac{1}{3} & \quad \times \quad \frac{3}{3} \\
\frac{2}{6} & \quad \times \\
\frac{1}{6} & \quad =
\end{align*}
\]

Make a rule:

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Renaming Fractions

Choose a card to match the fraction provided. Use the picture to help rename the fraction. Write the multiplication rule that renames the fraction.

Rename as

Legs

Place card here

\[
\begin{align*}
\frac{1}{2} &= \frac{1 \times 4}{8} \\
\frac{1}{4} &= \frac{1 \times 8}{8} \\
\frac{3}{4} &= \frac{3 \times 2}{8} \\
\frac{1}{8} &= \frac{1 \times 8}{8}
\end{align*}
\]

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Renaming Fractions

Choose a card to match the fraction provided.
Use the picture to help rename the fraction.
Write the multiplication rule that renames the fraction.

Make a rule:

Use the same rule:

 Rename as

hours

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Renaming Fractions

Choose a card to match the fraction provided.
Use the picture to help rename the fraction.
Write the multiplication rule that renames the fraction.

Place card here

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Script:
Teacher: “Use the same type of card to show me 1/3 and 1/6. Place them on your paper”
After all students respond by selecting two cards...
Teacher: “Sam, you’re right. What kind of cards did you select?”
Sam: “Ants.”
Have the students copy these fractions as shown under the heading Fraction Addition (below).

Now let’s rename these fractions to represent legs. Teacher: “How many legs does it take to make 1 whole ant? Show me your answers by holding up fingers.” Students should hold-up 6 fingers.

Rename each fraction to show the number of shaded legs. (Two legs out of six and one leg out of six respectively.)

How may shaded legs are there combined? (“Three out of six.”)

Fraction Addition

\[
\frac{1}{3} + \frac{1}{6} = \frac{3}{6}
\]

How many parts are shaded on both cards? 

How many parts are needed to make one whole?
Fraction Addition

How many parts are shaded on both cards?

How many parts are needed to make one whole?
Fraction Addition
Unlike Denominators

#1
\[
\frac{1}{2} + \frac{1}{3} = \frac{1}{2} + \frac{1}{3}
\]

How many parts are shaded on both cards?

How many parts are needed to make one whole?

#2
\[
\frac{2}{4} + \frac{3}{6} = \frac{2}{4} + \frac{3}{6}
\]

#3
\[
\frac{1}{2} + \frac{1}{3} = \frac{1}{2} + \frac{1}{3}
\]

#4
\[
\frac{2}{4} + \frac{3}{6} = \frac{2}{4} + \frac{3}{6}
\]
Fraction Addition
Unlike Denominators

#5

How many parts are shaded on both cards?

How many parts are needed to make one whole?

#6

#7

#8
Three Types of Fraction Addition/Subtraction Problems

**First:** Are the denominators the same?

- **Yes**
  - Type 1 Problem.
  - Use the same (common) denominator.
  - Add the numerators.

\[
\begin{array}{c}
\frac{2}{4} + \frac{1}{4} = \frac{3}{4}
\end{array}
\]

- **No**
  - Circle the larger denominator.

\[
\begin{array}{c}
\frac{2}{4} + \frac{1}{2} = \\
\frac{3}{4}
\end{array}
\]

**Second:** Is the larger (circled) denominator a multiple of the smaller?

- **Yes**
  - Type 2 Problem.
  - The larger (circled) denominator will be the new denominator for both fractions.
  - Rename the fractions, then add the numerators.

\[
\begin{array}{c}
\frac{2}{4} = \frac{2}{4} \\
\frac{1}{2} \times 2 = \frac{2}{4} \\
\frac{2}{4} + \frac{2}{4} = \frac{4}{4}
\end{array}
\]

- **No**
  - Multiply the two denominators.
  - Their product will be the new common denominator.
  - Rename both fractions, then add the numerators.

\[
\begin{array}{c}
\frac{2}{4} \times \frac{1}{3} = \frac{4}{12} \\
\frac{2}{4} = \frac{6}{12} \\
\frac{1}{3} = \frac{4}{12} \\
\frac{2}{4} + \frac{1}{3} = \frac{10}{12}
\end{array}
\]

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